Journal of Applied Mechanics and Technical Physics, Vol. 47, No. 1, pp. 68–70, 2006

FORCE ACTION OF A BODY WITH A PERMEABLE BOUNDARY ON A WALL IN A FLUID

V. L. Sennitskii

UDC 532.582

The ideal fluid flow due to fluid penetration through the boundary of an infinitely long solid cylinder in contact with a solid wall is determined. A formula is derived according to which the force exerted by a finite-length part of the cylinder on the wall is directed into the wall and can thus have an arbitrarily large absolute value.

Key words: *fluid, solid bodies in contact, permeable boundary, force action.*

1. The force interaction and the motion of an ideal or viscous fluid and a solid body located in it in the presence of a solid wall under time periodic actions on the hydromechanical system are considered in [1–4]. Such problems are of special interest if the distance between the solid body and the wall is not large compared to the size of the body, in particular, if the body in contact with the wall.

The present paper considers the following problem. In an ideal incompressible fluid bounded from outside by a flat absolutely solid wall there is a body Q (an absolutely solid, infinitely long circular cylinder of radius a) with a boundary permeable to the fluid (see Fig. 1). The wall and the body Q are at rest relative to the inertial rectangular coordinates X, Y, Z. The wall surface coincides with the plane $Y = 0$. The body Q is in contact with the wall along the straight line lying on the Z axis. The domain Ω occupied by the fluid outside the body Q is contained in a half-space $Y \geq 0$. On the side surface Γ of the body Q, the fluid performs prescribed motion at a velocity U in the direction of the outward normal to Γ. In the region Ω, the fluid flow is potential and plane. It is assumed that the fluid flow into the body Q is three-dimensional (the fluid flows into the body Q and flows out of it through its side surface and infinitely remote bases). It is required to determine the force F exerted on the wall in the Y direction by a body q (a part of the body Q) which is a circular cylinder of radius a with the base lying in the flow planes spaced a distance l apart.

2. We consider the fluid flow in the plane $Z = 0$.

The fluid velocity potential Φ satisfies the equation

$$\Delta\Phi = 0 \qquad (a < R < \infty, \quad 0 < Y < \infty) \tag{2.1}$$

and the conditions

$$\frac{\partial\Phi}{\partial Y} = 0 \quad \text{at} \quad Y = 0, \qquad -\infty < X < 0 \quad \text{and} \quad 0 < X < \infty; \tag{2.2}$$

$$\frac{\partial\Phi}{\partial R} = U \quad \text{at} \quad R = a, \qquad -\pi < \theta < \pi; \tag{2.3}$$

$$\nabla\Phi \to 0 \quad \text{at} \quad R \to \infty, \qquad 0 \leq Y. \tag{2.4}$$

Here R and θ are the polar coordinates linked to X and Y by the relations

$$X = -R\sin\theta, \qquad Y = a + R\cos\theta.$$

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; svovl@hydro.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 1, pp. 82–84, January–February, 2006. Original article submitted February 15, 2005.

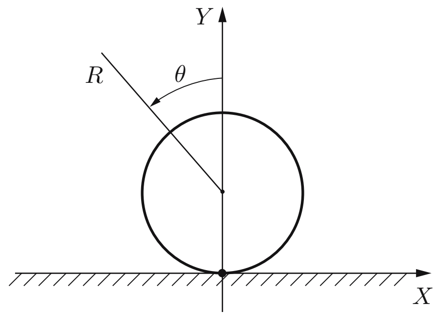

Fig. 1

We set

$$U = W\chi, \tag{2.5}$$

where W is a function of t;

$$\chi = 1 + \varepsilon^2 \frac{2(1+\cos\theta) - \varepsilon^2}{[2(1-\varepsilon)(1+\cos\theta)+\varepsilon^2][2(1+\varepsilon)(1+\cos\theta)+\varepsilon^2]} \tag{2.6}$$

($0 < \varepsilon < 2$ is a parameter).

We note that the dependence of W on t, in particular, can be periodic. According to (2.6) we have

$$\chi = 0 \quad \text{for} \quad \theta = \pm\pi; \tag{2.7}$$

$$\chi = 1 + O(\varepsilon^{2\alpha}) \quad \text{as} \quad \varepsilon \to 0, \qquad -\pi + \varepsilon^{1-\alpha} \le \theta \le \pi - \varepsilon^{1-\alpha}; \tag{2.8}$$

$$0 < \chi \le 9/8 + O(\varepsilon^2) \quad \text{as} \quad \varepsilon \to 0, \qquad -\pi < \theta < -\pi + \varepsilon^{1-\alpha} \quad \text{and} \quad \pi - \varepsilon^{1-\alpha} < \theta < \pi \tag{2.9}$$

($0 < \alpha < 1$ is a constant).

The dependence of χ on θ and ε specified by relation (2.6) is of interest because according to (2.5) and (2.7)–(2.9), the following holds:

(a) the velocity U vanishes at the point of contact of the circle $R = a$ with the X axis;

(b) for small values of ε everywhere on the circle $R = a$, except in a small neighborhood of the point of contact with the X axis, the velocity U can be regarded as a function of only t, and in the indicated neighborhood, the velocity U as a function of θ is bounded.

Problem (2.1)–(2.5) has a solution

$$\Phi = (1/2)aW \ln[R^2 + 2(1-\varepsilon)aR\cos\theta + (1-\varepsilon)^2a^2]$$

$$+ (1/2)aW \ln[R^2 + 2(1+\varepsilon)aR\cos\theta + (1+\varepsilon)^2a^2] + \varphi \tag{2.10}$$

(φ is a function of t), which represents the velocity potential of the fluid, whose flow is due to two fluid sources of intensity $2\pi aW$ located at the points $(0, -\varepsilon a)$ and $(0, \varepsilon a)$ on the plane $Z = 0$.

3. Momentum flows into the body q through its boundary.

The flux S of the Y-momentum of the fluid into the body q through its side surface is given by the formula

$$S = al\rho \int_{-\pi}^{\pi} \Big\{ \Big[\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 \Big] \cos\theta - \frac{\partial\Phi}{\partial Y}\frac{\partial\Phi}{\partial R} \Big\}\Big|_{R=a} d\theta, \tag{3.1}$$

where ρ is the fluid density.

We note that the force $-F$ is the flux of the Y momentum of the wall into the body q through its side surface.

We assume that the momentum flux into the body q through its bases is equal to zero and the center of mass of the body q is motionless. Then, the following relation should hold:

$$S - F = 0. \tag{3.2}$$

We determine the force F for small values of ε. Using (2.10), (3.1), and (3.2), we have

$$F = -\pi a l \rho W^2 \varepsilon^{-1}[1 + O(\varepsilon^{1/2})] \quad \text{as} \quad \varepsilon \to 0. \tag{3.3}$$

From (3.3), in particular, it follows that $F < 0$; i.e., the force exerted on the wall by the body q is directed into the wall (the body exerts a pressure on the wall), and in this case, $|F|$ can be arbitrarily large.

4. It is known that small gas bubbles in fluid move as solid spheres because of the presence of surface-active agents [5]. In view of this, the aforesaid [in particular, problem (2.1)–(2.5) and formula (3.3)] can be considered as a basis for the development of a mathematical model for the behavior of a hydromechanical system consisting of a fluid, a solid wall, and a gas bubble of time-dependent radius in contact with the wall.

We note that bubble radius can change with time, for example, periodically.

Let $\hat{a}$ and $\hat{W}$ be the characteristic values of the radius and the rate of change in the bubble radius. Then, the characteristic dimension of the force should be $\rho\hat{a}^2\hat{W}^2$. However, formula (3.3) indicates that because the bubble radius varies with time, the bubble can exert a force on the wall that is directed into the wall and have an absolute value larger than $\rho\hat{a}^2\hat{W}^2$. The action of this force should presumably result in a considerable local deformation or even failure of the wall.

REFERENCES

1. V. L. Sennitskii, "Motion of a circular cylinder in a vibrating fluid," *J. Appl. Mech. Tech. Phys.*, No. 5, 620–623 (1985).
2. B. A. Lugovtsov and V. L. Sennitskii, "Motion of a body in a vibrating fluid," *Dokl. Akad. Nauk SSSR.*, **289**, No. 2, 314–317 (1986).
3. V. L. Sennitskii, Motion of a sphere in a vibrating fluid in the presence of a wall," *J. Appl. Mech. Tech. Phys.*, **40**, No. 4, 662–668 (1999).
4. V. L. Sennitskii, "Force interaction of a sphere and a viscous fluid in the presence of a wall," *J. Appl. Mech. Tech. Phys.*, **41**, No. 1, 50–54 (2000).
5. V. G. Levich, *Physicochemical Hydrodynamics* [in Russian], Fizmatgiz., Moscow (1959).